COMBINED HEAT AND MASS TRANSFER FOR A PARTICLE IN A GAS FLOW WITH VARIABLE DEFINITIVE PARAMETERS

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Studies have been made [1-5] on linear stationary topics in mass and heat transfer for particles in a flow of incompressible liquid at low Peclet numbers. Similar nonlinear cases have been examined for any surface chemical kinetics in [6-9]. In [10] there is a study of the nonlinear joint heat and mass transfer at a spherical particle in a flow of compressible gas for the case of a power-law temperature dependence for the viscosity. In [11], mass and heat transfer were considered for a droplet and for a solid particle of any shape in transverse and shear flows of an incompressible liquid when the diffusion coefficient (thermal diffusivity) was arbitrarily dependent on concentration (temperature).

1. Formulation. New Variables. We consider the stationary heat and mass transfer for a particle (drop) of any shape in a translational flow of compressible gas, where the definitive parameters are dependent in any fashion on temperature. It is assumed that the concentration and temperature at the surface of the particle and far from it (at infinity) take constant values. We neglect thermal diffusion and barodiffusion and omit terms of the order of the square of the Mach number to write the equations in dimensionless variables as

$$\mathrm{Pe}_T\, c_p\rho(\mathbf{v}\cdot\operatorname{grad} T) = \operatorname{div}(\lambda \operatorname{grad} T); \tag{1.1}$$

$$\mathrm{Pe}_c\rho(\mathbf{v}\cdot\operatorname{grad} u) = \operatorname{div}(\rho\sigma \operatorname{grad} u); \tag{1.2}$$

$$r = r_s(\theta, \varphi),\ T = 1;\ r \to \infty,\ T \to 0; \qquad r = r_s(\theta, \varphi),\ u = 1;\ r \to \infty,\ u \to 0, \tag{1.3}$$

$$T = \frac{T_\infty - T_*}{T_\infty - T_s},\quad u = \frac{u_\infty - u_*}{u_\infty - u_s},\quad c_p = c_p(T) = \frac{c_{p*}(T_*)}{c_{p*}(T_\infty)},$$
$$\mathrm{Pe}_T = \frac{aU_\infty c_{p*}(T_\infty)\rho_\infty}{\lambda_*(T_\infty)},\quad \mathrm{Pe}_c = \frac{aU_\infty}{D(T_\infty)},\quad \rho = \frac{\rho_*}{\rho_\infty},$$
$$\lambda = \lambda(T) = \frac{\lambda_*(T_*)}{\lambda_*(T_\infty)},\quad \sigma = \sigma(T) = \frac{D(T_*)}{D(T_\infty)},\quad \lambda(0) = \sigma(0) = 1. \tag{1.4}$$

Here T_s, T_*, and T_∞ are the temperatures at the particle surface, in the gas flow, and the unperturbed value at infinity; u_s, u_*, and u_∞, relative (molar) concentrations at the particle surface, in the flow, and at infinity; c_{p*}, specific isobaric specific heat of the gas; ρ_* and ρ_∞, gas densities in the flow and at infinity; a, characteristic particle size (the radius for a sphere); U_∞, incident flow speed; λ_*, thermal conductivity of the gas; D, diffusion coefficient; $\mathbf{r} = (r, \theta, \varphi)$, spherical coordinate system immobilely linked to the particle (the angle θ is reckoned from the direction of the incident flow); $r = |\mathbf{r}|$, dimensionless polar radius referred to the characteristic particle size; Pe_T and Pe_c, thermal and diffusion Peclet numbers; and $r = r_s(\theta, \varphi)$, equation for the particle surface, where it is assumed that

$$T_s \neq T_\infty \ \text{and}\ u_s \neq u_\infty\ (\lambda > 0,\ \sigma > 0).$$

In writing (1.1)-(1.4) it has been assumed that the concentration of the minor component is small and does not influence the mean mass velocity, density, or temperature (in particular, the dependence of the transport coefficients on concentration is neglected).

We determine $\mathbf{v}$, ρ, and T from the solution for the flow of a viscous thermally conducting gas. In what follows we need in addition only the equation of continuity

$$\operatorname{div}(\rho\mathbf{v}) = 0, \tag{1.5}$$

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and the condition that the normal component of the gas velocity at the surface of the drop or particle is zero (contact condition)

$$r = r_s,\ (\mathbf{v}\cdot\mathbf{n}) = 0. \tag{1.6}$$

Here and subsequently, the notation is abbreviated by omitting the arguments θ and φ in $r_s \equiv r_s(\theta, \varphi)$ while **n** is the unit vector along the normal to the surface of the particle.

We also assume that the gas density is a known function of temperature:

$$\rho = \rho(T) \tag{1.7}$$

which is equivalent, for example, to the following: The viscosity is dependent only on temperature $\mu = \mu(T_*)$ and the Schmidt number is constant. With a view to generality, the explicit form of (1.7) will not at present be specified; some detailed examples are considered in Sec. 4.

For convenience in analysis, by analogy with [11] we replace the temperature T by a new auxiliary function

$$\Phi = \Phi(\lambda, T) = \int_0^T \lambda(\xi)\, d\xi \quad (\Phi(\lambda, 0) = 0,\ \Phi'_T(\lambda, 0) = 1). \tag{1.8}$$

Then we can reformulate the boundary-value problem of (1.1)-(1.4) on the basis of the identities

$$c_p\rho(\mathbf{v}\cdot\operatorname{grad} T) \equiv \operatorname{div}(\rho\mathbf{v}h) - h\operatorname{div}(\rho\mathbf{v}),\quad h(T) = \int_0^T c_p(\xi)\, d\xi,$$
$$\rho(\mathbf{v}\cdot\operatorname{grad} u) \equiv \operatorname{div}(\rho\mathbf{v}u) - u\operatorname{div}(\rho\mathbf{v})$$

together with (1.5) as follows in terms of the function Φ:

$$\mathrm{Pe}_T \operatorname{div}(\rho\mathbf{v}h) = \Delta\Phi;\ r = r_s,\ \Phi = J(\lambda);\ r \to \infty,\ \Phi \to 0; \tag{1.9}$$
$$\mathrm{Pe}_c\operatorname{div}(\rho\mathbf{v}u) = \operatorname{div}(\omega\operatorname{grad} u);\ r = r_s,\ u = 1;\ r \to \infty,\ u \to 0, \tag{1.10}$$
$$h = h(\Phi) \equiv h(T(\Phi)),\ \omega = \omega(\Phi) \equiv \rho(T(\Phi))\sigma(T(\Phi)).$$

Here $T = T(\Phi)$ is determined by reversing the function of (1.8), and in the linear case $\lambda = 1$ we have $T = \Phi$, while $J(\lambda)$ is given by

$$J(\lambda) = \int_0^1 \lambda(\xi)\, d\xi. \tag{1.11}$$

The purpose of the study is to derive the average Nusselt and Sherwood numbers, which are basic characteristics of the heat and mass transfer:

$$\mathrm{Nu}(\lambda, \mathrm{Pe}_T) = -\frac{1}{4\pi}\int_S \lambda(T)\frac{\partial T}{\partial n}\, dS = -\frac{1}{4\pi}\int_S \frac{\partial\Phi}{\partial n}\, dS, \tag{1.12}$$
$$\mathrm{Sh}(\omega, \mathrm{Pe}) = -\frac{1}{4\pi}\int_S \omega(\Phi)\frac{\partial u}{\partial n}\, dS,$$

where ∂/∂n is the derivative along the exterior normal to the surface of the particle $S = \{r = r_s(\theta, \varphi)\}$; $\mathrm{Sh}(\omega, \mathrm{Pe}) \equiv \mathrm{Sh}(\omega, \mathrm{Pe}_c, \mathrm{Pe}_T)$.

2. Solution Method. Auxiliary Equations. We further assume that the Peclet numbers are small and of the same order of magnitude:

$$\mathrm{Re}_\infty \to 0,\ \mathrm{Pe}_T = \mathrm{Re}_\infty\mathrm{Pr}_\infty,\ \mathrm{Pe}_c = \mathrm{Re}_\infty\mathrm{Sc}_\infty,$$
$$\mathrm{Pr}_\infty = O(1),\ \mathrm{Sc}_\infty = O(1), \tag{2.1}$$

$$\mathrm{Re} = \frac{aU_\infty\rho_*}{\mu}, \quad \mathrm{Pr} = \frac{\mu c_{p*}}{\lambda_*}, \quad \mathrm{Sc} = \frac{\mu}{\rho_* D}.$$

Here Re, Pr, and Sc are the Reynolds, Prandtl, and Schmidt numbers; the subscript ∞ corresponds to the unperturbed values of these parameters at infinity.

The solution to (1.1)-(1.4) or to (1.8) and (1.9) is derived by linking asymptotic expansions in terms of the small parameter Re_∞ [1-11]. The entire flow region is split into two subregions: the internal one $\Omega_1 = \{r_S \le r \le O(\mathrm{Re}^{-1}_\infty)\}$ and the external one $\Omega_\infty = \{O(\mathrm{Re}^{-1}_\infty) \le r\}$. As usual, we introduce the compressed coordinate $z = \mathrm{Re}_\infty r$ in the external region and the solution in each of the subregions is sought separately in the form of the internal expansion

$$\Phi = \Phi_0 + \mathrm{Pe}_T\Phi_1 + o(\mathrm{Re}_\infty), \quad u = u_0 + \mathrm{Pe}_c u_1 + o(\mathrm{Re}_\infty), \qquad (2.2)$$
$$r_s \leqslant r \leqslant O(\mathrm{Re}_\infty^{-1}),$$
$$\Phi_i = \Phi_i(r, \theta, \varphi), \ u_i = u_i(r, \theta, \varphi), \ i = 0, 1$$

and the external one

$$\Phi = \Phi^{(0)} + \mathrm{Pe}_T\Phi^{(1)} + o(\mathrm{Re}_\infty), \quad u = u^{(0)}\,\mathrm{Pe}_c u^{(1)} + o(\mathrm{Re}_\infty), \quad O(\mathrm{Re}_\infty^{-1}) \leqslant r, \qquad (2.3)$$
$$\Phi^{(0)} = u^{(0)} = 0, \ \Phi^{(1)} = \Phi^{(1)}(z, \theta, \varphi), \ u^{(1)} = u^{(1)}(z, \theta, \varphi), \ z = \mathrm{Re}_\infty r.$$

Here and subsequently the expansion is directly in terms of Pe_T and Pe_c for convenience.

The asymptotic solution in the internal region Ω_1 is constructed from the boundary conditions at the surface, while that in the external region Ω_∞ is derived from the boundary conditions at infinity; the unknown constants are determined by means of a linkage procedure [1-11].

The new variable of (1.7) means that all terms in the internal asymptotic expansion Φ_i and the external one $\Phi^{(i)}$ for the initially nonlinear boundary-value problem for the temperature of (1.1) and (1.3) satisfy linear equations in regions Ω_1 and Ω_∞ [11].

We substitute the representations of (2.2) for Φ and u into (1.8) and (1.9), and put $\mathrm{Pe}_T = \mathrm{Pe}_c = 0$ to get that the zero terms in the internal expansion are determined by solving the following equations with the boundary conditions at the surface of the particle:

$$\Delta\Phi_0 = 0; \ r = r_s, \ \Phi_0 = J(\lambda); \ r \to \infty, \ \Phi_0 \to 0; \qquad (2.4)$$
$$\mathrm{div}(\omega(\Phi_0)\,\mathrm{grad}\, u_0) = 0; \ r = r_s, \ u_0 = 1; \ r \to \infty, \ u_0 \to 0. \qquad (2.5)$$

The boundary conditions at infinity in (2.4) and (2.5) are obtained from the conditions for linkup with the zero terms in the external expansion of (2.3).

The solutions to (2.4) and (2.5) can be expressed in terms of the solution to a very simple auxiliary linear problem for an ordinary Laplace equation:

$$\Delta c_0 = 0; \quad r = r_s, \ c_0 = 1; \ r \to \infty, \ c_0 \to 0, \qquad (2.6)$$

which is familiar for particles of various shapes. In particular, $c_0 = r^{-1}$ for a spherical particle.

We choose the reference point for the radius vector **r** suitably to write the expression for c_0 of (2.6) in the following form [2], which tends to zero for $r \to \infty$, as for a harmonic function:

$$c_0 = \mathrm{Sh}(1, 0)r^{-1} + O(r^{-3}), \qquad (2.7)$$

where Sh(1, 0) is the mean Sherwood number corresponding to the mass transfer to the particle in an immobile immiscible liquid with $\rho = 1$ in the case of a constant diffusion coefficient $\sigma = 1$ in (2.6).

The solution to (2.4) can be put as

$$\Phi_0 = J(\lambda)c_0. \qquad (2.8)$$

We seek the solution to (2.5) as

$$u_0 = f(\Phi_0)f^{-1}(J(\lambda)),\ f(0) = 0, \tag{2.9}$$

where the function f is determined by substituting this expression into (2.5) and is then compared with the equation for the temperature distribution of (2.4). The comparison gives

$$f(\Phi_0) = \int_0^{\Phi_0} \frac{d\Phi}{\omega(\Phi)} \int_0^{T_0} \frac{\lambda(T)\,dT}{\rho(T)\,\sigma(T)} = \Phi\left(\frac{\lambda}{\rho\sigma},\ T_0\right),\quad \Phi_0 = \Phi(\lambda,\ T_0), \tag{2.10}$$

where for clarity we have also written the corresponding expression in terms of the zero term in the internal temperature expansion T_0.

The average Nusselt and Sherwood numbers of (1.12), which are determined by the zero terms in the internal expansion of (2.6)-(2.10), take the form

$$\mathrm{Nu}(\lambda,\ 0) = J(\lambda)\,\mathrm{Sh}(1,\ 0),\quad \mathrm{Sh}(\omega,\ 0) = J(\lambda)\,J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\mathrm{Sh}(1,\ 0). \tag{2.11}$$

The equality $f(J(\lambda)) = J(\lambda/\rho\sigma)$ has been used in deriving this formula.

We derive the first terms in the external expansion of (2.3) from the following limiting properties of the functions defining (1.9) and (1.10) for $r \to \infty$:

$$\rho \to 1,\ \lambda \to 1,\ \sigma \to 1,\ \omega \to 1,\ T \to \Phi,\ h \to \Phi,\ \mathbf{v} \to \mathbf{i}.$$

Here $\mathbf{i}$ is the unit vector parallel to the unperturbed velocity vector at infinity.

The equations for $\Phi^{(1)}$ and $u^{(1)}$ take the form

$$\begin{gathered}(\mathbf{i}\cdot \mathrm{grad}_{z_T}\Phi^{(1)}) = \Delta_{z_T}\Phi^{(1)},\quad z_T \to \infty,\quad \Phi^{(1)} \to 0,\\ (\mathbf{i}\cdot \mathrm{grad}_{z_c}u^{(1)}) = \Delta_{z_c}u^{(1)},\quad z_c \to \infty,\quad u^{(1)} \to 0,\\ z_T = \mathrm{Pe}_T r,\quad z_c = \mathrm{Pe}_c r.\end{gathered} \tag{2.12}$$

These equations coincide apart from the symbols with the analogous equations for the linear case [2]. Therefore, the solutions to (2.12) that satisfy the condition for linkup with the zero terms in (2.6)-(2.10) can be put as

$$\begin{gathered}\Phi^{(1)} = \mathrm{Nu}(\lambda,\ 0)\,z_T^{-1}\exp\left[\frac{1}{2}z_T(\eta - 1)\right],\quad \eta = \frac{(\mathbf{i}\cdot\mathbf{r})}{r} = \cos\theta,\\ u^{(1)} = \mathrm{Sh}(\omega,\ 0)\,z_c^{-1}\exp\left[\frac{1}{2}z_c(\eta - 1)\right].\end{gathered} \tag{2.13}$$

The second terms in the expansion of these expressions as series in the small quantities z_T and z_c define the following boundary conditions at infinity for the first terms in the internal asymptotic expansion by virtue of the representation of (2.3) and the linkup conditions:

$$r \to \infty,\ \Phi_1 \to (1/2)\mathrm{Nu}(\lambda,\ 0)(\eta - 1),\ u_1 \to (1/2)\mathrm{Sh}(\omega,\ 0)(\eta - 1). \tag{2.14}$$

Following [11], it can be shown that the following are the equations and boundary conditions correctly describing the first two terms in the internal expansion of (2.2):

$$\begin{gathered}\mathrm{Pe}_T\,\mathrm{div}(\rho\mathbf{v}h)_0 = \Delta\Phi,\ \Phi = \Phi_0 + \mathrm{Pe}_T\,\Phi_1,\\ r = r_s,\ \Phi = J(\lambda);\ r \to \infty,\ \Phi \to (1/2)\mathrm{Pe}_T\,\mathrm{Nu}(\lambda,\ 0)(\eta - 1);\end{gathered} \tag{2.15}$$

$$\begin{gathered}\mathrm{Pe}_c\mathrm{div}(\rho\mathbf{v}u)_0 = \mathrm{div}(\omega(\Phi)\mathrm{grad}\ u),\ u = u_0 + \mathrm{Pe}_c u_1,\\ r = r_s,\ u = 1;\ r \to \infty,\ u \to (1/2)\mathrm{Pe}_c\mathrm{Sh}(\omega,\ 0)(\eta - 1).\end{gathered} \tag{2.16}$$

The boundary-value problems in (2.15) and (2.16) have been written with accuracy to $o(\mathrm{Re}_\infty)$, as can be seen by comparing the equations and boundary conditions of (1.9) and (1.10) with those of (2.15) and (2.16) and using (2.2), (2.4), (2.5), and (2.14); the subscript zero on the left side in (2.15) and (2.16) corresponds to quantities into which have been substituted the zero terms from the internal expansion for Φ_0 and u_0.

We now seek the solution to the inhomogeneous equations (2.15) and (2.16) directly for Φ and u.

The solution to the thermal part of (2.15) is sought as the sum

$$\Phi = \overline{\Phi} + \delta\Phi, \tag{2.17}$$

where the terms satisfy the following equations and boundary conditions:

$$\Delta\overline{\Phi} = 0;\ r = r_s,\ \overline{\Phi} = J(\lambda);\ r \to \infty,\ \overline{\Phi} \to -(1/2)\mathrm{Pe}_T\mathrm{Nu}(\lambda, 0); \tag{2.18}$$

$$\Delta\delta\Phi = \mathrm{Pe}_T\mathrm{div}(\rho\mathbf{v}h)_0; \tag{2.19}$$
$$r = r_s,\ \delta\Phi = 0;\ r \to \infty,\ \delta\Phi \to (1/2)\mathrm{Pe}_T\mathrm{Nu}(\lambda, 0)\eta.$$

We seek the distribution of the relative concentration in the form

$$u = \overline{u} + \delta u, \tag{2.20}$$

where the terms are solutions to the following boundary-value problems:

$$\mathrm{div}(\omega(\overline{\Phi})\mathrm{grad}\,\overline{u}) = 0; \tag{2.21}$$
$$r = r_s,\ \overline{u} = 1;\ r \to \infty,\ \overline{u} \to -(1/2)\mathrm{Pe}_c\mathrm{Sh}(\omega, 0);$$

$$\mathrm{div}(\omega(\Phi)\mathrm{grad}\,\delta u) = -\mathrm{div}\{[\omega(\Phi) - \omega(\overline{\Phi})]\mathrm{grad}\,\overline{u}\} + \tag{2.22}$$
$$+ \mathrm{Pe}_c\mathrm{div}(\rho\mathbf{v}u)_0;\ r = r_s,\ \delta u = 0;\ r \to \infty,\ \delta u \to$$
$$\to (1/2)\mathrm{Pe}_c\mathrm{Sh}(\omega, 0)\eta.$$

It follows from (2.18), (2.19), (2.21), and (2.22) that $\delta\Phi = O(\mathrm{Re}_\infty)$, $\delta u = O(\mathrm{Re}_\infty)$.

A direct check shows that the solution to (2.18) can be represented in terms of the c_0 of (2.6) and (2.7) as

$$\overline{\Phi} = [J(\lambda) + (1/2)\mathrm{Pe}_T\,\mathrm{Nu}(\lambda, 0)]c_0 - (1/2)\mathrm{Pe}_T\,\mathrm{Nu}(\lambda, 0). \tag{2.23}$$

We seek the solution to (2.21) as

$$\overline{u} = Af(\overline{\Phi}) + B,\quad f(\overline{\Phi}) = \int\limits_0^{\overline{\Phi}} \frac{d\Phi}{\omega(\Phi)}. \tag{2.24}$$

As the $\overline{\Phi}$ of (2.18) is harmonic for any values of A and B, the expression of (2.24) is the solution to (2.21). The explicit form of the constants A and B is determined by solving the following linear algebraic system:

$$1 = Af(J(\lambda)) + B,\quad -\frac{1}{2}\mathrm{Pe}_c\,\mathrm{Sh}(\omega, 0) = Af\left(-\frac{1}{2}\mathrm{Pe}_T\,\mathrm{Nu}(\lambda, 0)\right) + B, \tag{2.25}$$

which is a consequence of the boundary conditions at the surface and at infinity for $\overline{\Phi}$ and $\overline{u}$ of (2.18) and (2.21). We restrict ourselves to the main term in the expansion of f for $\mathrm{Pe}_T \to 0$ [it is not necessary to incorporate the other terms, since the initial system of (2.15) and (2.16) has accuracy of only $o(\mathrm{Re}_\infty)$] in the second equation of (2.25) and solve this system, which on the basis of (2.11) gives us the following expressions for the coefficients:

$$A = J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\left[1 + \frac{1}{2}(\mathrm{Pe}_c - \mathrm{Pe}_T)J(\lambda)J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\mathrm{Sh}(1, 0)\right] + o(\mathrm{Re}_\infty), \tag{2.26}$$
$$B = -\frac{1}{2}(\mathrm{Pe}_c - \mathrm{Pe}_T)J(\lambda)J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\mathrm{Sh}(1, 0)\,o(\mathrm{Re}_\infty).$$

3. Term Interpretation. Mean Nusselt and Sherwood Numbers. Before we analyze (2.19) and (2.22), we consider the special case of a spherical droplet or particle ($r_s \equiv 1$), which enables us to interpret $\overline{\Phi}$, $\overline{u}$, $\delta\Phi$, and δu.

In accordance with the formula [8, 9]

$$\langle w\rangle = \frac{1}{4\pi r^2}\int\limits_{\Sigma_r} w\,d\Sigma = \frac{1}{4\pi}\int\limits_{-1}^{1}\int\limits_{0}^{2\pi} w(r, \eta, \varphi)\,d\varphi\,d\eta \quad (\eta = \cos\theta) \tag{3.1}$$

we introduce the surface-averaging operator, where Σ_r is the surface of a sphere of radius r.

For any function w dependent only on the radial coordinate r we have $\langle w(r)\rangle = w(r)$, and the averaging operator of (3.1) commutates with the operator for differentiation with respect to r. Also, in the present case of a spherical particle, the solution is independent of the φ coordinate $(\partial/\partial\varphi = 0)$, while the zero terms in the internal expansions of (2.2) are dependent only on r by virtue of the symmetry in (2.4) and (2.5); $\Phi_0 = \Phi_0(r) = J(\lambda)r^{-1}$, $u_0 = u_0(r)$. As the representation of (2.2) applies for Φ and u, the following formula applies to $o(\mathrm{Re}_\infty)$ for any analytic function G:

$$\langle G(\Phi, u)\rangle = G(\langle\Phi\rangle, \langle u\rangle), \tag{3.2}$$

which can be proved by direct check.

We use these features and follow [8, 9] in integrating the equations and boundary conditions of (2.15) and (2.16) with respect to φ and η within the same limits as in (3.1) to get for the means that

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\langle\Phi\rangle = 0; \tag{3.3}$$

$$r = 1,\ \langle\Phi\rangle = J(\lambda);\ r \to \infty,\ \langle\Phi\rangle \to -(1/2)\mathrm{Pe}_T\mathrm{Nu}(\lambda, 0);$$

$$\frac{1}{r^2}\frac{d}{dr}r^2\omega(\langle\Phi\rangle)\frac{d}{dr}\langle u\rangle = 0; \tag{3.4}$$

$$r = 1,\ \langle u\rangle = 1;\ r \to \infty,\ \langle u\rangle \to -(1/2)\mathrm{Pe}_c\mathrm{Sh}(\omega, 0).$$

In deriving (3.3) and (3.4) we have used the results $\langle\eta\rangle = 0$, $\langle\rho v_r\rangle = 0$, of which the second is a consequence of (1.5) and the absence of flow through the surface of the particle of (1.6).

The mean Nusselt and Sherwood numbers of (1.12) are defined by the following to $o(\mathrm{Re}_\infty)$:

$$\mathrm{Nu} = -\left[\frac{d\langle\Phi\rangle}{dr}\right]_{r=1},\quad \mathrm{Sh} = -\left[\omega(\langle\Phi\rangle)\frac{d\langle u\rangle}{dr}\right]_{r=1}. \tag{3.5}$$

Comparison of the equations and boundary conditions of (2.18) and (2.21) with those of (3.3) and (3.4) shows that we have as follows for a spherical particle:

$$\bar{\Phi} = \langle\Phi\rangle,\ \delta\Phi = \Phi - \langle\Phi\rangle;\ \bar{u} = \langle u\rangle,\ \delta u = u - \langle u\rangle. \tag{3.6}$$

Formulas (3.6) mean that $\bar{\Phi}$ and $\bar{u}$ are surface means, while $\delta\Phi$ and δu are the deviations of the initial quantities Φ and u from their mean values.

This comparison, together with (3.5), shows that the second terms $\delta\Phi$ and δu of (2.17) and (2.20) do not make any contribution to the integral heat and material fluxes to the particle surface. This in turn means that the expansions of $\delta\Phi$ and δu for $r \to \infty$ do not contain a source term proportional to r^{-1}. This assertion can be proved by integrating (2.19) and (2.22) over a reference gas volume V enclosed between the surface S of a particle and the surface of a sphere Σ_R of radius R entirely enclosing the particle. We transfer from the volume integral to a surface one (with respect to S and Σ_R) in accordance with Gauss's formula and pass to the limit $R \to \infty$ to get that the surface integrals of the right parts of (2.19) and (2.22) become zero because of the features of the zero terms in Φ_0 and u_0, the no-flow condition of (1.6), and the boundary conditions at the surface. It also follows that there is no source term in the expansion of $\delta\Phi$ and δu for $r \to \infty$ because one of the remaining two surface integrals becomes zero, namely that over the surface of the sphere S these integrals correspond to the left sides of (2.19) and (2.22).

One can consider $\delta\Phi$ and δu similarly for the general case of a particle of any shape. For this we consider a family of surfaces c_0 = const (in the case of a spherical particle, this family consists of surfaces of concentric spheres of constant radius r = const by virtue of $c_0 = r^{-1}$). By virtue of (2.23) and (2.24) we also have $\bar{\Phi}$ = const and $\bar{u}$ = const at these surfaces. The surface corresponds to $c_0 = 1$, while far from the particle the surface c_0 = const by virtue of (2.7) tends asymptotically to spherical form for $r \to \infty$. Equations (2.19) and (2.22) forget the shape of the particle far from it (i.e., the structure of the solution for $r \to \infty$ will be as for a spherical particle) and, therefore, the asymptotic expansions of $\delta\Phi$ and δu should not contain source terms proportional to r^{-1} for $r \to \infty$. On this basis, we integrate (2.19) and (2.22) over the control volume V and transfer to surface integrals with respect to S and Σ_R. We apply an argument analogous to that above to conclude that $\delta\Phi$ and δu do not contribute to the mean Nusselt and Sherwood numbers, as in the case of a spherical particle. This means that $\delta\Phi$ and δu can be treated as fluctuations in Φ and u about the mean values $\bar{\Phi}$ and $\bar{u}$ at the surfaces c_0 = const.

We use (2.23), (2.24), and (2.26) for the mean Nusselt and Sherwood numbers to get to $o(\mathrm{Re}_\infty)$ that

$$\mathrm{Nu}(\lambda, \mathrm{Pe}_T) = J(\lambda)\,\mathrm{Sh}(1, 0)\left[1 + \frac{1}{2}\mathrm{Pe}_T\,\mathrm{Sh}(1, 0)\right],$$
$$\mathrm{Sh}(\omega, \mathrm{Pe}) = \mathrm{Sh}(\omega, 0)\left[1 + \frac{1}{2}\mathrm{Pe}_T\,\mathrm{Sh}(1, 0) + \frac{1}{2}(\mathrm{Pe}_c - \mathrm{Pe}_T)\right.$$
$$\left.\times \mathrm{Sh}(\omega, 0)\right] = \mathrm{Nu}(\lambda, \mathrm{Pe}_T)\,J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\left[1 + \frac{1}{2}(\mathrm{Pe}_c - \mathrm{Pe}_T)\,\mathrm{Sh}(\omega, 0)\right], \tag{3.7}$$
$$\mathrm{Sh}(\omega, 0) = J(\lambda)\,J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\mathrm{Sh}(1, 0).$$

Then (2.1) enables us to write (3.7) as

$$\mathrm{Nu}(\lambda, \mathrm{Re}_\infty) = J(\lambda)\,\mathrm{Sh}(1, 0)\left[1 + \frac{1}{2}\mathrm{Re}_\infty\,\mathrm{Pr}_\infty\,\mathrm{Sh}(1, 0)\right] + o(\mathrm{Re}_\infty), \tag{3.8}$$

$$\mathrm{Sh}(\omega, \mathrm{Re}_\infty) = J(\lambda)\,J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\mathrm{Sh}(1, 0)\left\{1 + \frac{1}{2}\mathrm{Re}_\infty\,\mathrm{Sh}(1, 0)\quad\left[\mathrm{Pr}_\infty + (\mathrm{Sc}_\infty - \mathrm{Pr}_\infty)\,J(\lambda)\,J^{-1}\left(\frac{\lambda}{\rho\sigma}\right)\right]\right\} + o(\mathrm{Re}_\infty).$$

From (3.8) we see that when the Lewis–Semenov number Le = $\mathrm{Pr}_\infty/\mathrm{Sc}_\infty$ = 1 (with this accuracy), there is an analogy between the heat and mass transfer at the particle.

<u>Note.</u> The first formula in (3.7) can be derived directly from (2.15) as in [11]. For this purpose we multiply both parts of (2.15) by Φ_0 and integrate over the control volume of the gas V on the basis of

$$\Phi_0\Delta\Phi \equiv \mathrm{div}\,(\Phi_0\,\mathrm{grad}\,\Phi) - \mathrm{div}\,(\Phi\,\mathrm{grad}\,\Phi_0) + \Phi\Delta\Phi_0,$$
$$\Phi_0(c_p\rho\mathbf{v}\cdot\mathrm{grad}\,T)_0 = \Phi_0\,\mathrm{div}\,(\rho\mathbf{v}h)_0 \equiv \mathrm{div}\,(\rho\mathbf{v}\zeta) - \zeta\,\mathrm{div}\,(\rho\mathbf{v}),$$
$$\zeta = \zeta(\Phi_0) = \int\limits_0^{T_0(\Phi_0)} c_p(\xi)\,\Phi(\lambda, \xi)\,d\xi,$$

whose latter terms become zero because the function Φ_0 of (2.4) is harmonic and by virtue of the equation of continuity (1.5); here the function $T_0(\Phi_0)$ is obtained by inverting the expression $\Phi_0 = \Phi(\lambda, T_0)$ of (1.8). We use Gauss's formula to convert to surface integrals to get finally that

$$\sum_{j=1}^{6} I_j = 0,\quad I_1 = -\int\limits_S \Phi_0\frac{\partial\Phi}{\partial n}\,dS,\quad I_2 = \int\limits_S \Phi\frac{\partial\Phi_0}{\partial n}\,dS, \tag{3.9}$$
$$I_3 = \mathrm{Pe}_T\int\limits_S \rho\zeta(\Phi_0)(\mathbf{v}\cdot\mathbf{n})\,dS,\quad I_4 = \int\limits_{\Sigma_R}\Phi_0\frac{\partial\Phi}{\partial n}\,d\Sigma_R,$$
$$I_5 = -\int\limits_{\Sigma_R}\Phi\frac{\partial\Phi_0}{\partial n}\,d\Sigma_R,\quad I_6 = -\mathrm{Pe}_T\int\limits_{\Sigma_R}\rho\zeta(\Phi_0)(\mathbf{v}\cdot\mathbf{n})\,d\Sigma_R.$$

To calculate the first three integrals in (3.9) we use the boundary conditions of (2.4) and (2.15), the definition of the mean Nusselt number of (1.12), and the condition (1.6) for the absence of gas flow through the surface. To calculate the latter three integrals, we use the representations of (2.7) and (2.8) for Φ_0, the boundary condition at infinity of (2.15) for Φ, and the formulas $d\Sigma_R = O(R^2)$, $\zeta(\Phi_0) \simeq \Phi_0^2/2 = O(R^{-2})$, $\mathbf{v} \simeq \mathbf{v}$ which apply for large R. On this basis we allow the radius R of the sphere to tend to infinity to get the following expressions for the integrals:

$$I_1 = 4\pi J(\lambda)\,\mathrm{Nu}(\lambda, \mathrm{Pe}_T),\ I_2 = -4\pi J(\lambda)\,\mathrm{Nu}(\lambda, 0), \tag{3.10}$$
$$I_3 = I_4 = I_6 = 0,\ I_5 = -2\pi J(\lambda)\,\mathrm{Nu}(\lambda, 0)\,\mathrm{Sh}(1, 0).$$

We now use (2.11), (3.9), and (3.10) to get the first formula of (3.7).

4. Power-Law Dependence of Viscosity on Temperature. Particles of Various Shapes. For a particle of a given shape, one can determine the rate of the convective heat and mass transfer in accordance with (3.7) and (3.8) by calculating the integrals $J(\lambda)$, $J(\lambda/\rho\sigma)$ of (1.11) and determining the mean Sherwood number Sh(1, 0) corresponding to the solution to the linear auxiliary problem for the Laplace equation of (2.6).

We first derive the expressions for the integrals J for certain typical cases.

We consider a power-law dependence of the viscosity on temperature:

$$\mu(T_*) = \mu_0 T_*^n, \quad \mu_0 = \text{const.} \tag{4.1}$$

Then, as in [10], it is assumed that the gas has a constant specific heat and that Pr and Sc are constant. Then (4.1), together with these assumptions, corresponds to the following functions that define the problem of (1.1)-(1.4):

$$\lambda(T) = \omega(T) = \rho\sigma = [1 + (T_s/T_\infty - 1)T]^n.$$

As the mean Nusselt and Sherwood numbers are given by (3.7) or (3.8) for a power-law temperature dependence for the viscosity, we have

$$J(\lambda) = \frac{1}{n+1}\left[\frac{1-(T_s/T_\infty)^{n+1}}{1-(T_s/T_\infty)}\right], \quad J\left(\frac{\lambda}{\rho\sigma}\right) = 1. \tag{4.2}$$

Formulas (3.8) and (4.2) were derived in [10] apart from the difference in symbols and normalization for the case of a spherical particle Sh(1, 0) = 1.

We now examine the effects of compressibility on the convective heat and mass transfer. We assume that the thermal conductivity and diffusion coefficient are constant and independent of temperature:

$$\lambda = \sigma = 1. \tag{4.3}$$

Also, the pressure in the flow differs only slightly from the unperturbed pressure at infinity if the gas velocity is low (differences of the order of the square of the Mach number). This means that the equation of state $p_* = \rho_* RT_*$ (where p_* is pressure and R is the gas constant) can be modified by replacing p_* by $p_\infty = \rho_\infty RT_\infty$, which gives us the following expression after conversion to a dimensionless variable for the density:

$$\rho = \rho(T) = [1 + (T_s/T_\infty - 1)T]^{-1}. \tag{4.4}$$

It follows from (4.3) and (4.4) that the mean Nusselt and Sherwood numbers are defined by (3.7), where

$$J(\lambda) = 1, \quad J\left(\frac{\lambda}{\rho\sigma}\right) = \frac{1}{2}\left(1 + \frac{T_s}{T_\infty}\right).$$

We formulate the ratio of this mean Sherwood number to the auxiliary Sherwood number corresponding to the case of an incompressible fluid:

$$\frac{\mathrm{Sh}(\omega, 0)\,|_{\rho=\rho(T)}}{\mathrm{Sh}(\omega, 0)\,|_{\rho=1}} = \frac{2T_\infty}{T_s + T_\infty}.$$

This formula shows that the compressibility reduces the rate of mass transfer at a hot particle for $T_s > T_\infty$ (by comparison with the analogous process in an incompressible fluid) and increases the mass transfer for a cold one for $T_s < T_\infty$.

We now give some detailed values for Sh(1, 0) for aspheric particles [Sh(1, 0) = 1 for a sphere].

The following formula applies for a thin circular disk [2]:

$$\mathrm{Sh}(1, 0) = 2/\pi \approx 0.637.$$

Consider a particle as an ellipsoid of rotation with semiaxes a and b, where a is the equatorial radius and b is the polar radius, which lies along the axis of rotation. We take the equatorial radius a as the characteristic length scale and get Sh(1, 0) as [12]

$$\mathrm{Sh}(1, 0) = \begin{cases} (\chi^2+1)^{-1/2}[\operatorname{arcctg}\chi]^{-1}, & a \geqslant b, \\ (\chi^2-1)^{-1/2}[\operatorname{arcth}\chi]^{-1}, & a \leqslant b, \end{cases}$$

$$\chi = \left|\left(\frac{a}{b}\right)^2 - 1\right|^{-1/2}, \quad \operatorname{arcth}\chi = \frac{1}{2}\ln\left(\frac{\chi+1}{\chi-1}\right).$$

We now consider a solid dumbbell particle consisting of two contacting spheres of radii a_1 and a_2. Then Sh(1, 0) is given by [13]

$$\mathrm{Sh}(1, 0) = -\frac{a_2}{a_1+a_2}\left[\psi\left(\frac{a_1}{a_1+a_2}\right) + \psi\left(\frac{a_2}{a_1+a_2}\right) + 2\gamma\right], \quad (4.5)$$

$$\psi(x) = \frac{d}{dx}\ln\Gamma(x) \quad \left(\lim_{x\to 0} x\psi(x) = -1, \quad \psi\left(\frac{1}{2}\right) = -\gamma - 2\ln 2, \quad \psi(1) = -\gamma\right).$$

Here the radius a_1 of the first sphere has been taken as the characteristic length scale, while $\psi = \psi(x)$ is the logarithmic derivative of a gamma function, $\Gamma = \Gamma(x)$ is the gamma function, and $\gamma = 0.577215...$ is Euler's constant.

In the particular case of a particle consisting of two contacting spheres of equal radius $a_1 = a_2 = a$, from (4.5) we get

$$\mathrm{Sh}(1, 0) = 2\ln 2 \approx 1{,}386.$$

Note. A difference from the case of an incompressible fluid [11] is that in (3.7) and (3.8) we cannot add terms proportional to $\mathrm{Pe}^2 \ln \mathrm{Pe}$ since, in the case of a viscous compressible thermally conducting gas we do not know the corresponding asymptotic expansion for the velocity field far from the particle (naturally, apart from the main term **i**). Therefore, in order to derive the subsequent terms in the asymptotic expansion with respect to the small Reynolds number one has to examine the complete problem, which involves incorporating the equations of motion as well as (1.1), (1.2), and (1.5).

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